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Dynamics of mechanical stress in active elastic thin sheets and application to biofilm morphogenesis

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What is a biofilm?

- Surface attached community of microorganisms
- Cells are embedded in extracellular matrix (ECM)
- E.g. dental plaque

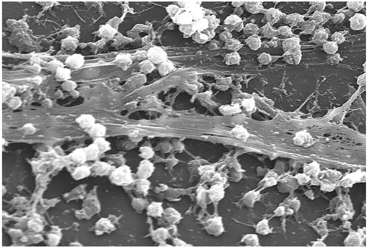


Figure: *Staphyloccocus aureus* biofilm

Lab grown biofilms

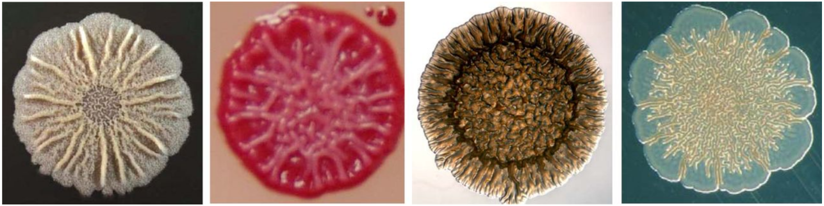


Figure: Left to right: *B. subtilis*, *Agrobacterium tumefaciens*, *Vibrio fischeri*, *Pseudomonas fluorescens*

Experimental detail: coffee ring effect

- Artefact of lab-grown biofilms

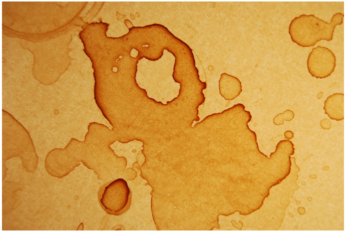


Figure: Coffee stain effect

Stress and bioluminescence

- Bioluminescence as a response to shear stress
- Movie

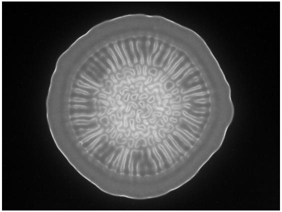


Figure: Lab grown *Vibrio harveyi* biofilm

Modelling biofilms

Assumptions

- Thin: z scale $<<$ x or y scales
- Macroscopic - individual cells not considered

Model

- Embedded surface
- Harmonic and bending forces
- Growth coupled to harmonic force

Theory of 1D thin elastic sheets

- Curve in 2d plane
- Reference state - length L, arclength param. s
- Deformed state - length $\hat{L}$, arclength param. $\hat{s}$
- Reparametrise by angle $\phi(s)$ and strain $\gamma(s) = \frac{d\hat{s}}{ds}$
- Energy functional

$$E_{1D} = \int_0^L \left[\frac{B}{2} \left(\frac{d\phi}{ds} \right)^2 + \frac{Y}{2} (\gamma - 1)^2 \right] ds + W_{ext}$$

Equilibria of 1D thin elastic sheets

- No tension: $\gamma = 1$

$$B\frac{d^2\phi}{ds^2} + p\sin\phi = 0$$

- With tension

$$B\frac{d}{d\hat{s}}\left(\gamma\frac{d\phi}{d\hat{s}}\right) - \frac{dV}{d\phi} = 0$$

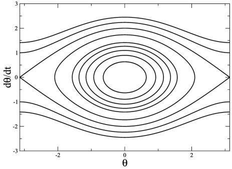


Figure: Phase portrait for nonlinear pendulum

Theory of 2D thin elastic sheets

- Function $z = z(x, y)$
- $\gamma(s) \rightarrow \epsilon_{\alpha\beta}(x, y) = g_{\alpha\beta} - \bar{g}_{\alpha\beta}$
- $\phi(s) \rightarrow \phi_{\alpha\beta}(x, y)$ related to curvature
- $\sigma_{\alpha\beta} = \sigma_{\alpha\beta}(\epsilon_{\alpha\beta}, \phi_{\alpha\beta})$ constitutive equation
- Energy functional

$$E_{2D} = E_s + E_b + W_{ext} = \frac{1}{2}\int_{\mathcal{A}} [\sigma_{\alpha\beta}\epsilon_{\alpha\beta} + M_{\alpha\beta}\phi_{\alpha\beta}] d\mathcal{A} + W_{ext}$$

Simulation of 2D thin elastic sheets

- Mesh - triangulation
- Brownian dynamics
- Growth
- Coffee ring

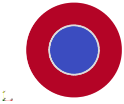


Figure: Mesh with different edge types assigned

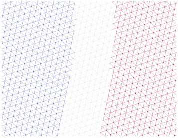


Figure: Zoomed-in version of the mesh

Stability and timescales

- Elastic timescale
- Growth timescale
- Integration timestep
- Mesh fineness
- Movie

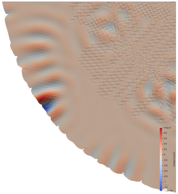
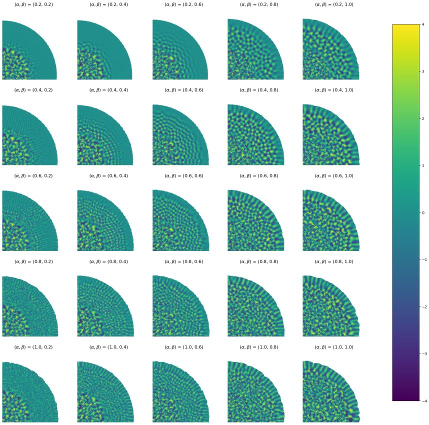


Figure: Average length:radius = 0.6:50

Stretching and bending stiffness



Stretching and bending stiffness

- Energy functional for circular string in linearised regime

$$E = \int_0^L \frac{1}{2} T \left(\frac{dy}{dx}\right)^2 + \frac{1}{2} B \left(\frac{d^2y}{dx^2}\right)^2 dx$$

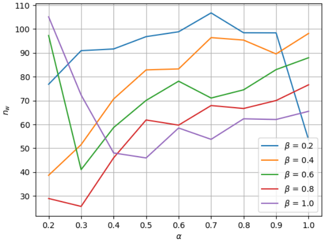


Figure: Number of wrinkles for varying α, β

$\sigma_{\theta z}$ shear stress

- Shear component of interest
- Expected relation to z component

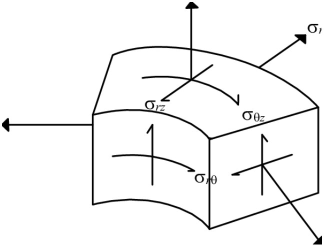


Figure: Stress tensor in cylindrical coordinates

Stages of formation

- Movie

1. In-plane stressing
2. Stress relief
3. Growth

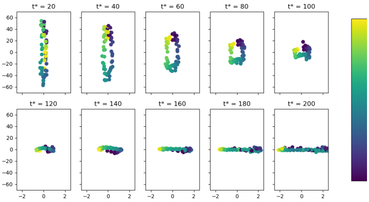


Figure: $\sigma_{\theta z}$ against z for different times

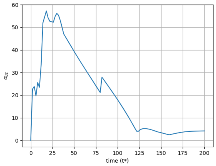


Figure: Stress dissipation

Wrinkle analysis

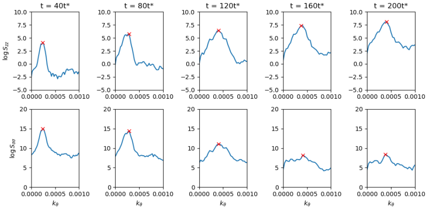


Figure: log power spectrum

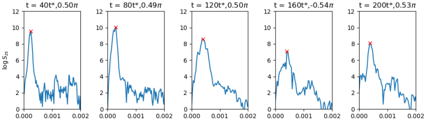


Figure: log Cross-spectrum

Figure: log power spectra from Welch's method

Conclusion

- Patterns emerge from energy minimisation
- Growth drives transition
- Pattern formation occurs in a short timescale
- Ways forward

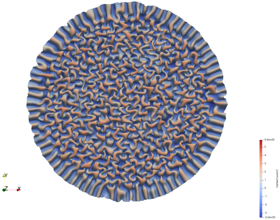


Figure: Without substrate

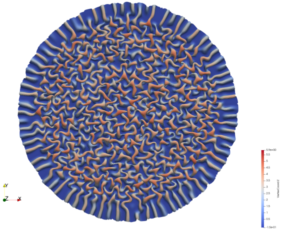


Figure: With substrate

References

- Oshri, O 2017, ‘Pattern Formation in Thin Elastic Sheets’, PhD thesis, Tel Aviv University
- D. A. Matoz-Fernandez, Fordyce A. Davidson, Nicola R. Stanley-Wall, and Rastko Sknepnek, Phys. Rev. Research 2, 013165 (2020)