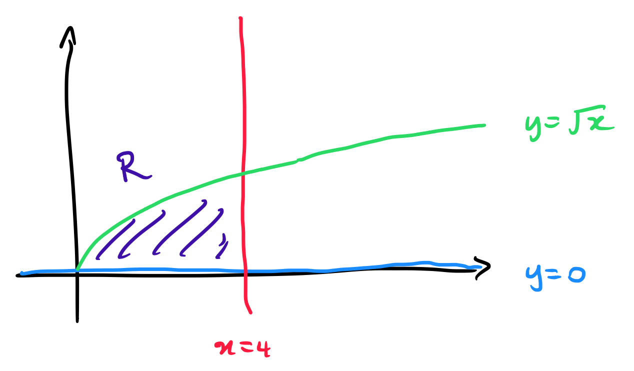
4d) REGION R:



R IS THE REGION WITH $0 \le x \le 4$, $0 \le y \le \sqrt{x}$.

BOUND OF y DEPENDS ON x $\Rightarrow$ y INTEGRAL ON RIGHT.

INTEGRAL:

$$\iint_R f(x,y)\,dA = \int_{x=0}^{4}\int_{y=0}^{\sqrt{x}} \frac{y}{1+x}\,dy\,dx$$

$$= \int_{x=0}^{4}\left[\frac{\frac{1}{2}y^2}{1+x}\right]_{y=0}^{\sqrt{x}} dx$$

$$= \int_{x=0}^{4} \frac{1}{2}\,\frac{x}{1+x}\,dx$$

$$= \frac{1}{2}\int_0^4 \frac{1+x-1}{1+x}\,dx$$

$$= \frac{1}{2}\int_0^4 1-\frac{1}{1+x}\,dx$$

$$= \frac{1}{2}\Big[x-\log(1+x)\Big]_0^4 \qquad (\log \text{ IS NATURAL LOG}).$$

$$= \frac{1}{2}\big[4-\log 5\big].$$

6.) a) $\sum_{k=1}^{3}\frac{1}{k} = 1+\frac{1}{2}+\frac{1}{3}$

$= \frac{11}{6}$

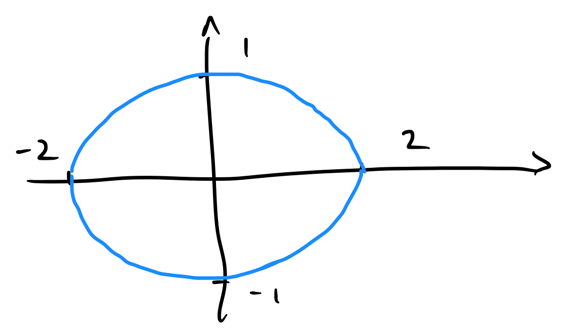
b) $\sum_{k=1}^{n} 1 = \underbrace{1+\dots+1}_{n \text{ TIMES}}$

$= n.$

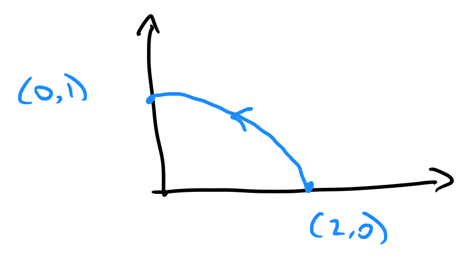
11b) $x^2+4y^2=4$ IS THE EQUATION FOR AN ELLIPSE.

SINCE $(2,0)$, $(0,1)$ AND E.G. $(-2,0)$ LIE ON THE ELLIPSE,

WE KNOW IT IS THE ELLIPSE



AND SPECIFICALLY C IS THE ARC



I'LL GIVE 3 OPTIONS:

A) TRIGONOMETRIC: $(x(t), y(t)) = (2\cos(t), \sin(t))$, $0 \le t \le \frac{\pi}{2}$.

B) BY y: USE $y=t$.

THEN $x^2+4y^2=4$

$\Rightarrow x^2 = 4(1-y^2)$

$\Rightarrow x(t) = 2\sqrt{1-t^2}$

SO $(x(t), y(t)) = (2\sqrt{1-t^2}, t)$, $0 \le t \le 1$.

C) BY x: USE $x=t$.

THEN $x^2+4y^2=4$

$\Rightarrow y^2 = \frac{4-x^2}{4}$

$\Rightarrow y(t) = \sqrt{1-\frac{t^2}{4}}$

BUT THIS HAS INCORRECT ORIENTATION.

ORIGINALLY, t GOES FROM 0 TO 2. SWITCH TO

$x=2-t$, AND

$y = \sqrt{1-\frac{(2-t)^2}{4}} = \sqrt{\frac{4t-t^2}{4}}$

$(x(t), y(t)) = \left(2-t, \sqrt{\frac{4t-t^2}{4}}\right).$

12d). PARAMETRISE C:

A PARAMETRISATION IS GIVEN BY

$$\vec{p}(t) = (1-t)\begin{pmatrix}1\\0\\2\end{pmatrix} + t\begin{pmatrix}3\\4\\1\end{pmatrix}$$

$$= \begin{pmatrix}2t+1\\4t\\2-t\end{pmatrix}. \qquad \text{WITH } 0 \le t \le 1.$$

$$\dot{\vec{p}}(t) = \begin{pmatrix}2\\4\\-1\end{pmatrix}.$$

THEN

$$\int_C \vec{g}(\vec{x})\cdot d\vec{x} = \int_0^1 \vec{g}(\vec{p}(t))\cdot\dot{\vec{p}}(t)\,dt$$

$$= \int_0^1 \begin{pmatrix}2(2t+1)4t\\(2t+1)^2+(2-t)\\4t\end{pmatrix}\cdot\begin{pmatrix}2\\4\\-1\end{pmatrix} dt$$

$$= \int_0^1 16t(2t+1)+4\big((2t+1)^2+(2-t)\big)-4t\,dt$$

$$= 4\int_0^1 8t^2+4t+4t^2+4t+1+2-t-t\,dt$$

$$= 4\int_0^1 12t^2+6t+3\,dt$$

$$= 4\Big[4t^3+3t^2+3t\Big]_0^1$$

$$= 40.$$